WIND DATA SUiTE Report Series

WIND DATA SUiTE Report No. 2018-01

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Citation:

Frey, H. (2018): Determining atmospheric stratification by the vertical profile of wind speeds. WIND DATA SUiTE, Germany, Report No. 2018-01, 2 pp.
https://www.WindDataSuite.de/wds/publics/WDS-Report201801-DrHF-SI.pdf

DETERMINING ATMOSPHERIC STRATIFICATION BY THE VERTICAL PROFILE OF WIND SPEEDS

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Summary

A new analysis method for determining atmospheric stratification by the shape of the vertical profile of horizontal wind speeds is presented. It is shown that this method is quite reliable though not considering any air temperature data.

1. Introduction

It is well known that the vertical profile of horizontal wind speeds (this is called the **"wind profile"** in the following) has a logarithmic shape above ground when averaging the wind speeds over a long-term time range. The logarithmic shape, however, is modified by stable and unstable atmospheric stratification (air density stratification, this is called **"stratification"** in the following) to some extent, especially when considering shorter time ranges or distinct time steps.
The logarithmic wind profile according to the surface boundary layer theory is given by the well-known equation

$$(1) \quad v(z) = \frac{u_*}{\kappa}\left(\ln\left(\frac{z-D}{z_0}\right) - \Phi\left(\frac{z-D}{L}\right)\right)$$

where z is the height above ground, $v(z)$ the horizontal wind speed in the height z, u_* the friction velocity, $\kappa = 0.4$ the Von-Karman constant, D the displacement height above ground, z_0 the roughness length, Φ the empirical stability function for unstable, neutral ($\Phi = 0$), and stable stratification, and L the Monin-Obukhov length. For neutral stratification, equation (1) can be reduced to

$$(2) \quad v(z) = \frac{u_*}{\kappa}\ln\left(\frac{z-D}{z_0}\right)$$

Many approaches (e.g. [1]), more or less empirical, do exist so far in order to determine the Monin-Obukhov length for unstable and stable stratification from measurements, and all of these, however, are more or less unsatisfying. The reason for these difficulties simply arise from the fact, that a local wind profile at a certain time is not the result of a simple one-dimensional vertical cause-and-effect process as presupposed by the surface boundary layer theory and the Monin-Obukhov similarity theory.
So, the aim is not to determine the Monin-Obukhov length by evaluating any air potential temperatures but to determine stratification by the shape of the wind profile itself.

2. The Analysis Method

An idealized example for the different wind profiles under the different stratification conditions is depicted in Fig.1. The analysis method is based on the fact that on the ln(z)-v plane (ln(z) is the y-axis, v is the x-axis), the shape of the logarithmic wind profile will be exponential for unstable stratification, will be linear for neutral stratification, and will be logarithmic for stable stratification [2], as depicted in Fig.2.

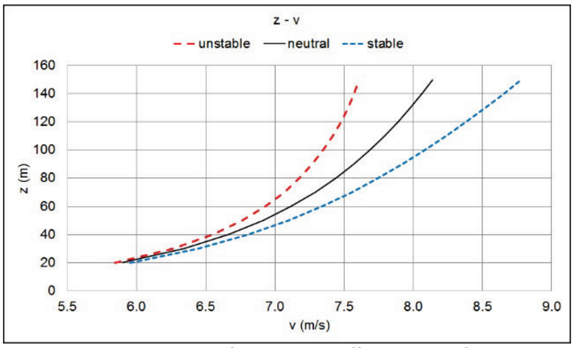

Fig.1: The wind profiles under different stratification conditions

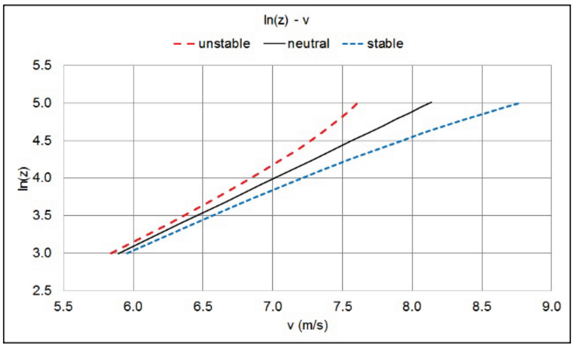

Fig.2: As in Fig.1 but for the ln(z)-v plane

So, for each wind profile, an exponential regression, a linear regression, and a logarithmic regression will be performed by examining the equation

$$(3) \quad \ln z = f(v(z))$$

All three regressions can be transferred to a linear regression of the form

$$(4) \quad y = a \cdot x + b$$

with a suitable variable substitution. The exponential regression $\ln(z) = be^{av(z)}$ yields

$$(5) \quad \ln(\ln(z)) = av(z) + \ln(b)$$
$$x = v(z)$$
$$y = \ln(\ln(z))$$

The linear regression $\ln(z) = av(z) + b$ yields:

$$(6) \quad x = v(z)$$
$$y = \ln(z)$$

The logarithmic regression $\ln(z) = a\ln(v(z)) + b$ yields

(7) $x = \ln(v(z))$
$y = \ln(z)$

For an example, these regressions have been calculated for the idealized wind profiles shown in the figures above. The resulting coefficients of determination, r^2 (which are the squared correlation coefficients r), are listed in Tab.1.

	unstable	neutral	stable
exponential	0.9995	0.9944	0.9807
linear	0.9932	1.0000	0.9956
logarithmic	0.9859	0.9978	0.9998

Tab.1: The coefficients of determination

In fact, the maximum r^2 shows that on the ln(z)-v plane the exponential regression is the best fit for the profile with unstable stratification, the linear regression is the best fit for the profile with neutral stratification, and the logarithmic regression is the best fit for the profile with stable stratification.

Before using a measured wind profile for such a regression analysis, it has to fulfil several stability conditions:

- The number of measurement heights must be at least three ($K \geq 3$, this excludes $r = 1$ due to $K = 2$).
- The correlation coefficient must be positive definite ($r > 0$, this excludes wind profiles with wind speeds decreasing with heights) and finite ($r \leq 1$, this excludes wind profiles with an almost constant wind speed over all heights).
- The correlation coefficient must pass a Student t-test for its 0.05-quantile ($t_{empirical} > t_{critical}$, this guarantees statistical significance).
- The wind speed difference of two adjacent heights must be greater than -0.5 m/s ($v(k) - v(k-1) > -0.5\,m/s$, this excludes wind profiles with too strong air column instabilities).

So-called "stratification indices", SI, will then be calculated by certain functions of the deviations of the resulting r^2 values of the exponential from the linear regression for unstable stratification, and of the logarithmic from the linear regression for stable stratification, respectively, such that the SI values will be greater than zero for stable stratification, zero for neutral stratification, and less than zero for unstable stratification. Standardization guarantees that

(8) $\sigma_{|SI|} \approx 1$

where $\sigma_{|SI|}$ is the standard deviation of all $|SI|$ values with $|SI| > 0$.
Thus, the SI values not only reflect the type of the stratification (stable, neutral, unstable) but also its intensity.

3. Results

The stratification indices, SI, have been calculated for a wind measurement comprising a time range of one year and a height range from 50 m to 200 m with 31 height levels.

Then, from the calculated SI time series values, the mean diurnal variation of SI has been calculated. This is shown in Fig.3.

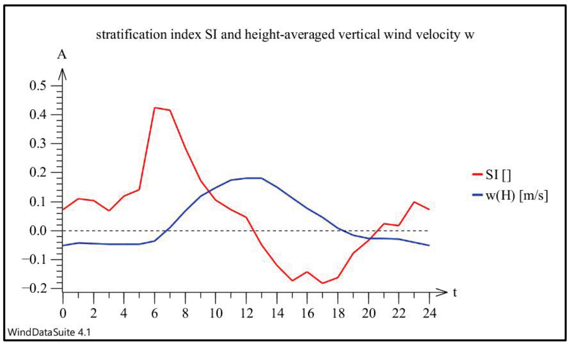

Fig.3: The mean diurnal variation of SI and of the height-averaged vertical wind velocity w(H)

As clearly can be seen in Fig.3, the stratification changes with a diurnal cycle. The stable stratification, which has been established during the night due to nighttime cooling, starts to get eroded with the beginning of insolation in the morning. The stratification changes more and more from stable to neutral before noon, then changes more and more from neutral to unstable after noon due to the continuous insolation. The stratification reaches its most unstable state at the evening, and with the ending of the insolation in the evening, it starts to get more and more stable again due to the nighttime cooling, whereby it takes some time (until approx. 21 hours) to dissipate the unstable stratification.
For comparison and validation, the mean diurnal variation of the height-averaged vertical wind velocity component, $w(H)$, has been calculated and is also depicted in Fig.3. The diurnal cycle of $w(H)$ correlates quite well with the diurnal cycle of SI. A correlation calculation yields a maximal correlation coefficient of $r = 0.86$ at a phase shift of SI of +6 hours = +π/2 of the diurnal cycle.

4. Conclusions

Calculating stratification indices, SI, by applying regression analysis to the wind profiles as here described yields very good results with respect to the resulting type and intensity of the stratification.

This here described diagnostic technique is very well suited for analysing stratification conditions during a wind measurement but, of course, is far from being usable for any prognostic model calculations.

5. References

[1] Ha, K.-J.et al. (2007): Evaluation of Boundary Layer Similarity Theory for Stable Conditions in CASES-99. Monthly Weather Review, Vol.135, 3474-3483.
[2] Kraus, H. (2008): Grundlagen der Grenzschicht-Meteorologie: Einführung in die Physik der Atmosphärischen Grenzschicht und in die Mikrometeorologie. Springer Verlag, 2008, 211 pp.